DOCUMENT RESUME

ED 159 082 SE 024 966

AUTHOR Dockweiler, Clarence J.
TITLE Using Systems of Reference with Spatial Concepts: Their Development from 5th Grade to College Age.
PUB DATE [78]
NOTE 22p.; Not available in hard copy due to marginal legibility of original document

EDRS PRICE MF-$0.83 Plus Postage. HC Not Available from EDRS.
DESCRIPTORS *Cognitive Development; *Cognitive Tests; *Educational Research; Elementary Secondary Education; Higher Education; *Mathematics Education; *Spatial Relationship
IDENTIFIERS *Piagetian Tasks; Research Reports; *Spatial Concepts

ABSTRACT

Students in grades 5, 8, 11, and college were administered Piagetian tasks dealing with horizontals and verticals. The tasks were the bottle task, the pendulum task, and the faucet task. The results disagreed with Piaget's suggested timetable. Since the horizontal task results did not yield a majority of correct predictions for any of the groups, this research does not conclusively establish that a majority of individuals ever attain this ability. A follow-up interview validated the data and also provided evidence that the use of coordinate systems of reference in the plane develops prior to the ability to use spatial reference systems. (Author/MP)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED159082

Using Systems of Reference with Spatial Concepts: Their Development from 5th Grade to College Age

U S DEPARTMENT OF HEALTH, EDUCATION & WELFARE
NATIONAL INSTITUTE OF EDUCATION

THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL NATIONAL INSTITUTE OF EDUCATION POSITION OR POLICY

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY

Clarence J. Dockweiler

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC) AND USERS OF THE ERIC SYSTEM."

Clarence J. Dockweiler

Texas A&M University

Running Head: Using Systems of Reference with Spatial Concepts

SE 024 966

Abstract

Students of 5th, 8th, and 11th grades and college level students were administered Piagetian tasks dealing with horizontal and verticals. The results disagreed with Piaget's suggested timetable. Since the horizontal task results did not yield a majority predicting correctly for any of the groups, this research does not conclusively establish that a majority of individuals ever attain this ability.

A follow-up interview validated the data and also provided evidence that the use of coordinate systems of reference in the plane develops prior to the ability to use spatial reference systems.

Using Systems of Reference with Spatial Concepts: Their Development from 5th Grade to College Age

Piaget and Inhelder have a fascinating discussion of the possible sequence followed by children as they progress toward acquisition of systems of frames of reference in The Child's Conception of Space, (Piaget and Inhelder, 1956). Many examples of children's reactions to appropriate tasks are given. As a result, a developmental scheme is formulated which takes into account various levels during which children develop the ability to abstract various spatial concepts from environmental tasks.

In a water level task the scheme begins with very young children's (5 years or less) inability to distinguish water surfaces or planes. The first use of a system of reference occurs when a child is able to abstract the surface of a liquid as a plane. At this point the child may indicate that the water level in a tipped jug will remain parallel to the base of the jug. The child is using the jug as a system of reference.

This scheme progresses with the culmination being the ability to predict a water level of a tipped jug to be horizontal. Piaget and Inhelder report that this ability occurs at an average age of 9 years with a range between the ages of 6 years, 6 months and 12 years. These conclusions are summarized in the following quote:

> In short, the distinctive feature of this final

> substage is the general coordination of all angles and parallels throughout the entire field of objects under consideration. It is this process which enables the child to discover the physical constancy of the horizontal water level and the vertical plumb line, a concept which is eventually independent of experimental observation. (Piaget and Inhelder, 1956, p. 412)

Further experimentation and speculation lead Piaget and Inhelder to the conclusion that these systems of reference develop simultaneously with the development of an understanding of coordinate axes and their application to environmental tasks (p. 418). It would appear, therefore, that by the age of 12 almost all children have the ability to use an appropriate system of reference and apply a coordinate system to a spatial task, according to Piaget's scheme.

The focus of this research study was to attempt to verify student's ability to use reference systems and, secondarily, to see if the application and use of coordinate systems develop in a parallel way. Piaget seems to suggest that one is dependent on the other, that is, before a subject can properly apply rectangular coordinates he must first have the capability to use reference systems with regard to verticals and horizontals (p. 380). In any event, both capabilities should be established by the age of 12.

Piaget closely relates the use of systems of reference to his level of formal operational thinking. The following quotations include phrases which characterize the level of formal operational thought.

> During Substage IIIA it is possible to see the concepts of vertical and horizontal gradually crystallizing out in the actual course of the experiment. Only from the beginning of Substage IIIB, about the age of 9 are they applied logically and consistently to all situations right from the start of the interview. (p. 406)

> In the case of the most advanced children like Cue (11;1) and Tis (11;6) who are on the verge of abstract thought, we find that their system of co-ordinates has become virtually conventional or hypothetico-deductive. (p. 411)

Attainment of the use of systems of reference should, therefore, be one measure in determining if subjects have reached the level of formal operational thought.

Considerable research has been conducted which has attempted to verify Piaget's findings. Several studies have indicated that many (if not most) subjects of high school age and older tend to respond at the concrete level rather than at a formal level. (Lawson and Rennen, 1974; Dunlop and Fazio, 1976).

A study with Australian school children dealt specifically with water levels and horizontality (Sheppard, 1975). In testing children from 6 years of age to 11 years, his results showed that 63.45% of 11 year olds had achieved the concept of horizontality. Using Sheppard's criteria for concept attainment, a majority of the subjects had acquired this capability at the approximate time Piaget had predicted. Insofar as Sheppard's study measures achievement of formal reasoning, his reuslts differ dramatically from the results referred to earlier in which high school age and older students were judged to be at the concrete level.

These students are only illustrative of those which report seemingly inconsistent results. These inconsistencies may in part, be accounted for by the considerably different method used and by the different criteria used to determine concept attainment.

Description of Study

The study was conducted during the Spring Term of 1977. The set of subjects consisted of 375 students from the fifth grade, eighth grade, eleventh grade, and university pre-service teacher (PST). The ages of the students ranged from 11 to 12 years of age at the fifth grade level; to 20 to 22 years of age at the university level. The fifth grade level was chosen as the starting point since these children were at the approximate age for which Piaget has suggested an almost universal capability to perform the required tasks.

The subjects were given three paper and pencil reference system tasks. These tasks were administered to groups of students. The tasks were the "bottle" task, the "pendulum" task, and the "faucet" task. In the "bottle" task, the subject was shown a cut-away picture of a bottle partially filled with water, as in Figure 1. It was then explained to the subject that the jug was to be tilted at

Insert figure 1 about here

an angle in the bottom picture of the figure. The subject was asked to draw in the water line in the bottom picture, where he thought it should be.

After completion of the "bottle" task, the "pendulum" task was administered. In this task, a task card picturing a cut-away view of a box was shown, with a weight suspended by a cord from the top center as in Figure 1. In a similar way as with the "bottle" task, the subjects were asked to draw in the weight and string if the box was tilted as shown.

The "faucet" task was then administered as a consistency check for the "bottle" task. As figure 1 pictures, each subject was given a task card picturing a tilted jug to be filled from the faucet. The subject was then asked to draw the water line when the jug was filled to approximately half.

Responses to the three tasks were then classified for analyses. Two investigators independently classified the responses. A response

was judged to be correct if it fell within approximately 5° of horizontal or vertical. In the few instances where disagreements appeared, the tasks were studied again to reach agreement. Several categories were used to classify errors, but it is sufficient for this analysis to consider a response as correct or incorrect.

The second part of the data gathering consisted of short interviews with individual subjects chosen at random from each group. The total number interviewed approximated 25% of the subjects in the study. The interview session was divided into two parts. First, each subject was asked to repeat the "bottle" task. The subject was then given an opportunity to explain his placement of the water line. To further verify the subject's response, a series of questions and tasks were administered culminating with an actual bottle for those who had responded incorrectly or inconsistently. The responses were then classified according to their consistency and to the extent to which a rule was used.

The second phase of the interview consisted of a point location problem. The purpose of this task was to obtain data on the relationship of the development of the use of frames of reference and the ability to use coordinate systems. As previously cited, Piaget and Inhelder have indicated that these two abilities develop simultaneously or at least sequentially. The task required the subject to look at a piece of plain paper on which four dots had been randomly placed. The subject was asked to locate the four dots on

another piece of paper of the same size. In other words, if one paper would be placed over the other, the location of the dots would exactly coincide. The interviewer noted if the subject used a reference system or approximation in locating the dots. A reference system was judged to be used if measurements were obviously made. These measurements could be rectangular; normally that would mean a measurement from the top of the paper down to the point in question and also a measurement from the side of two paper. The measurement could also incorporate an angular measure or the use of polar-like coordinates. If the placement techniques was not obvious the interviewer would ask for an explanation of technique used. A ruler and protractor were available, but the subjects were not directed to them initially. Their suggested used was mentioned later in the interview.

Results

Group Results. Piagetian research has indicated that the ability to use frames of reference should be developed by the age of 12 years, more or less. The investigators anticipated some diviation from that figure, but did not expect the degree of deviation obtained. The results of the group administered tasks were surprisingly low, particularly with regard to the bottle and faucet tasks. The per cent figure in Table 2 show that less than 40% of

Insert Table 1 here

the 11th graders and the PST group were able to correctly predict water level. For these subjects, at an age which suggests attainment of adulthood, age does not bring with it a capability to use frames of reference with regard to horizontals.

The "pendulum" task was correctly performed by a much larger portion of each of the groups. Although it was not the purpose of this study to differentiate between the two task types, some evidence does seem to exist to suggest that horizontal and vertical capabilities are differentially developed. A research study designed to focus on this aspect would be necessary to more firmly establish this result.

Table 1 shows an increase in the percent of subjects for all these tasks from the 5th to the 11th groups. This does provide some evidence to support the idea that attainment of these capabilities is developmental. It also should be pointed out that for every task the increase in percent of subjects giving correct answers often with the 11th grade group; there is even an insignificant decrease from 11th to PST. This fact suggests a leveling off in the developmental growth.

Insert Table 2 here

A measure of the significance of the group difference was obtained by using a Chi-Square test (Table 2). Although a positive growth is apparent, none of the task results showed a significant

change from the 5th to 8th group. Whereas, in every instance the growth from 5th to 11th and from 8th to 11th was significant. A growth period during the early secondary school years after a period of insignificant growth is an interesting result.

For this study the growth seemed to peak with the 11th grade group. Reported changes from 11th to PST are insignificant. A significant increase in use of reference frames occurred from 5th to PST and also from 8th to PST with one exception; the change reported on the bottle task for the 8th grade group to the PST group was insignificant. An interpretive caution is raised at this point. Some care was taken to obtain a representative sample of students for each of the 5th, 8th and 11th grade group. The PST group, however, is a special subset of the college level population and is not necessarily representative since all subjects in the PST group were elementary education majors. Future research is being planned to have these tasks done across all colleges on a university campus in order to study a more representative group.

Interview Results. The purpose for conducting the interview was two-fold. First, the interviews were designed to verify the paper and pencils tasks during the group administration. Secondly, an additional task was administered to gather data on the student's ability to perform a point location task.

A sequence of activities was designed to give in-depth information

with regard to the subjects reaction to the "bottle" task. The comments and reaction of students were recorded and analyzed for rule use. That is, if the subject made a comment such as "the water should be level" or "it will be parallel to the table" the investigators concluded that a correct rule was present. The results were not conclusive, since only one subject from each of the 5th and 8th grade groups indicated a rule, while 12 of the 11th grade group and 8 of the PST group used a rule.

The main objective of conducting the follow-up interview was to verify the results of the group administered tasks. The information obtained in the interview regarding the "bottle" task was compared to the results of the "bottle" task. If the results were in agreement, either both correct or both incorrect, the results were judged to be consistent. Table 3 summarizes the results as percents. The group results were judged to be a reasonably accurate reflection of the actual response, since 75% or more of each group showed consistency from the group administration to the interview.

Insert Table 3 here

A secondary purpose of the interviewing process was to determine the capabilities of students with regard to the point location task. Those interviewed were given the point location task as described previously. The procedure of each student was observed to

ascertain the use of some frame of reference. The interviewers anticipated that the subjects using a reference system would use a variation of a rectangular coordinate system, an angular system (one which would use angle measurement), or a combination of the two. Table 4 summarizes the results of those interviewed. The

---

Insert Table 4 here

---

results point to a considerable difference in the capabilities of these students between the "bottle" task and this "dot" task. All but one of the students interviewed from the 11th and PST groups successfully used a reference system for the "dot" task. Whereas, less than 35% of each of these groups successfully performed the "bottle" task. (See Table 1) A similar difference in results may be observed for the 5th and 8th groups. These results would appear to be strong evidence in support of the contention that students do not develop the use of these two abilities at the same time. The point location task appeared to be a much easier problem to solve than the horizontal task. Piaget's suggestion that the development of the tasks is sequential; that is, from reference systems to coordinate systems, is not supported by this research. In fact, the sequence may be in the opposite direction!

Discussion

This study was an attempt to verify Piaget's results with reagard to the acquisition of the use of frames of reference. The results of this study disagree with the Piagetian research in terms of when these capabilities are acquired. Data was gathered from different age level groups starting with 5th grades (11-12 years of age) and continuing with groups from 8th grade, 11th grade, and college level. Not only did a majority of 5th graders fail to correctly complete the tasks, but none of the age levels showed a majority with an ability to perform the task dealing with horizontals. These results would seem to suggest that a large number of subjects may never be able to correctly use a frame of reference in a similar situation. Further research is needed to more carefully identify the point of acquisition of such a capability.

A second area of disagreement lies in the ability of students to apply a rectangular coordinate system as compared with the horizontal and vertical frame of reference. These results suggest that these capabilities are developed differentially and do not develop simultaneously or sequentially as Piaget's results suggest.

The results were rather startling to the investigators. It would appear that the implications are potentially far-reaching. Many research problems are possible and several have been mentioned. The following questions are raised to suggest additional areas.

1. How does the inability to students to use systems of reference relate to environmental models? That is, do norizontal ceiling lines, horizons, etc. communicate a "horizontal" concept?
2. Is there a greater, earlier exposure to rectangular coordinate systems that could introduce a "training" effect to upset a predicted developmental sequence?

References

Dunlop, David L. and Fazio, Frank. Piagetian Theory and absract preferences of college science students. Journal of College Science Teaching, 1976, 5 (5), 297-300.

Lawson, Anton E. and Pennen, John W. A quantitative analysis of responses to Piagetian tasks and its' implication for curriculum. Science Education, 1974, 58 (4), 545-559.

Piaget, Jean and Inhelder, Barbel. The Child's Conception of Space. London, Routledge and Paul, 1956.

Sheppard, John L. Children's performance on a test of the concept of horizontally, using water levels. The Australian Journal of Education, 1975, 19 (2), 191-192.

Table 1

Measure of Correct Responses

		Tasks		
Group	n	Bottle	Pendulum	Faucet
5th Grade	107	14.95%	44.34%	9.35%
8th Grade	85	18.82%	50.59%	16.47%
11th Grade	110	32.11%	70.00%	38.18%
PST	73	31.51%	69.86%	31.51%

Table 2

Chi-Square Group by Group Comparison

Group	Group 8th	11th	PST
Bottle Task			
5th	.51	8.80**	6.99**
8th		4.36*	3.40
11th			.01
Pendulum Task			
5th	.74	14.53**	11.36**
8th		7.64**	6.06*
11th			.00
Faucet Task			
5th	2.20	24.75***	14.24***
8th		11.04***	4.96*
11th			.84

* $p < .05$

** $p < .01$

*** $p < .001$

Table 3

Consistency of Group Results with

Interview Results

Group	Per Cent in Agreement
5th	79%
8th	83%
11th	75%
PST	95%

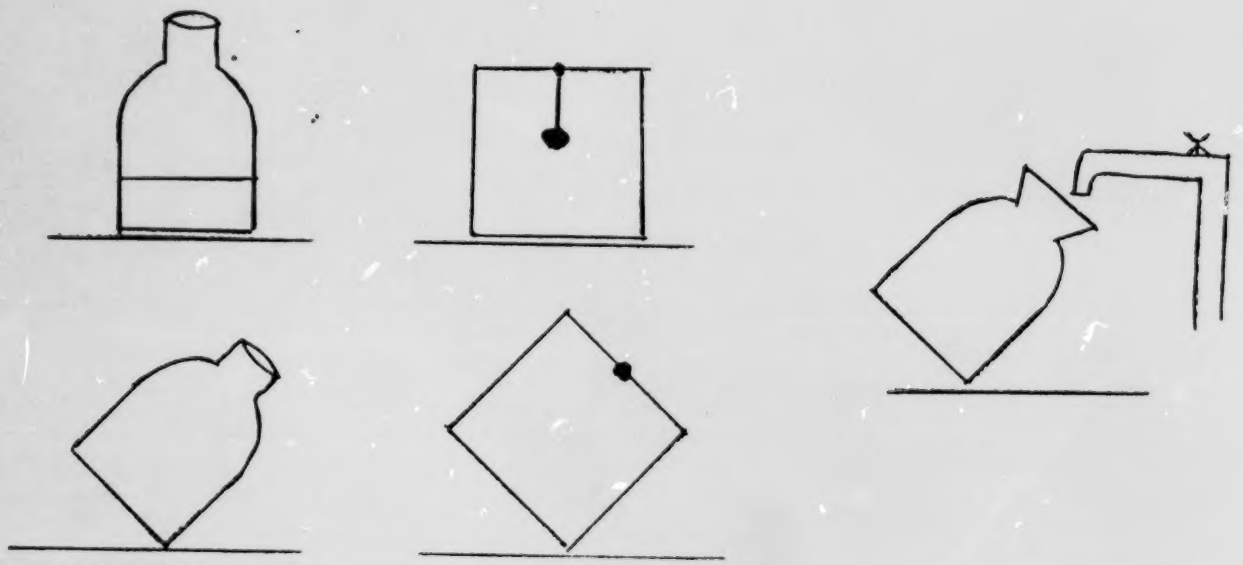
Table 4

Use of Reference System

in Dot Task

Group	n	Percent Using Reference System
5th	26	61.5%
8th	23	60.9%
11th	28	95.4%
PST	19	100%

Figure 1



"Bottle" task "Pendulum" task "Faucet" task